Application Note 1

Principles of Laser Heterodyne Measurements

The optical heterodyne method is commonly used in laser physics to compare the frequencies of two laser sources. The method takes advantage of readily available electronic test equipment, and is capable of producing very accurate results. Unlike wavelength comparisons, optical heterodyne measurements can be made in air without the need to make corrections for the index of refraction. In addition, high accuracy (<1 in 10^9) wavelength measurements generally require large apparatus, whereas the physical size of optical heterodyne systems can be quite compact regardless of the desired accuracy.

GENERATING A HETERODYNE SIGNAL

An optical heterodyne signal is generated by combining the output beams of two lasers onto the active area of a fast photodetector. If the photodetector has sufficient bandwidth, the electrical output will represent the frequency difference between the two lasers. This occurs because a photodiode responds to the square of the electric field (i. e. the intensity) of the incident light.

If the time-varying part of the electric field of a constant-amplitude, single-frequency laser is represented by:

$$E_1(t) = E_1 \cos \omega_1 t$$

the response of the photodetector can be written as:

$$\begin{aligned} I(t) &\propto E_1^2(t) \\ &= \frac{1}{2} E_1^2 \left[1 + \cos\left(2\omega_1 t\right) \right] \\ &\to \frac{1}{2} E_1^2 \quad as \quad \omega_1 \to \infty \end{aligned}$$

The high oscillatory term has been dropped since ω_1 is much larger than the detector bandwidth, leaving only a dc term.

If a second laser field, $E_2(t)$, is overlapped with the first, the response of the photodiode to the field $E(t)=E_1(t)+E_2(t)$ is:

$$\begin{aligned} I(t) &\propto E^2(t) \\ &= \left[E_1 \cos \omega_1 t + E_2 \cos \omega_2 t\right]^2 \\ &= \frac{1}{2}\left[E_1^2 + E_2^2 + E_1^2 \cos(2\omega_1 t) + E_2^2 \cos(2\omega_2 t)\right] \\ &\qquad + E_1 E_2 \left(\cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\right) \\ &\to \frac{1}{2}\left[E_1^2 + E_2^2\right] + E_1 E_2 \cos(\omega_1-\omega_2)t \quad as\ \omega_1, \omega_2 \to \infty \end{aligned}$$

The electrical output of the photodiode now contains two dc terms corresponding to the intensities of the two laser fields, plus a frequency difference term. This frequency difference term is referred to as the heterodyne signal.

ANALYZING THE HETERODYNE SIGNAL

The heterodyne signal contains complete information about the frequency stability of the laser sources. Analysis of the heterodyne signal can therefore reveal the characteristics of the laser sources. There are two common methods for defining frequency stability – one for the frequency domain and one for the time domain. Both have advantages and disadvantages; however, the time domain definition is most commonly used to characterize optical frequency standards.

Definition of Frequency Stability – Frequency Domain

The frequency domain definition for the frequency stability is the spectral density $S_y(f)$ of the instantaneous frequency fluctuations y(t), where we define the instantaneous fractional frequency deviation from the nominal frequency ν_0 as:

$$y(t) \equiv \frac{1}{2\pi\nu_0} \frac{d\varphi(t)}{dt}$$

and the heterodyne signal is given by:

$$V(t) = \left[V_0 + \varepsilon(t)\right] \sin\left[2\pi\nu_0 t + \varphi(t)\right]$$

The spectral density of the frequency fluctuations can be related to the spectral density of the phase fluctuations Sφ(f) by:

$$S_y(f) = \left(\frac{1}{\nu_0}\right)^2 f^2 S_\varphi(f)$$

Definition of Frequency Stability – Time Domain

The second definition of frequency stability is based on the sample variance of the fractional frequency fluctuations. We define the average fractional frequency deviation over the time interval from t_k to $t_k+\tau$ as:

$$\bar{y}_k = \frac{1}{\tau} \int_{t_k}^{t_k+\tau} y(t)dt$$

where τ is referred to as the sampling time, averaging time, or gate time. For N samples, each separated by a time interval T, we define a general N-sample variance by:

$$\left\langle \sigma_y^2(N,T,\tau) \right\rangle = \left\langle \frac{1}{N-1} \sum_{n=1}^{N} \left(\bar{y}_n - \frac{1}{N} \sum_{k=1}^{N} \bar{y}_k \right)^2 \right\rangle$$

where the angle brackets denote the infinite average over time. Because this equation does not converge as $N \to \infty$ for some noise processes, a 2-sample variance, or Allan Variance, is preferred:

$$\left\langle \sigma_y^2(N=2, T=\tau, \tau) \right\rangle \to \sigma_y^2(\tau) = \left\langle \frac{1}{2} \left(\bar{y}_{k+1} - \bar{y}_k \right)^2 \right\rangle$$

One can estimate $\sigma_y^2(\tau)$ from a finite number of samples, M, as:

$$\sigma_y^2(\tau) \cong \frac{1}{2(M-1)} \sum_{i=1}^{M-1} \left(\bar{y}_{i+1} - \bar{y}_i \right)^2$$

A simple way to generate $\sigma_y^2(\tau)$ is by counting the heterodyne signal with an electronic frequency counter for M successive periods of length τ. The data can be logged by a computer and $\sigma_y^2(\tau)$ easily calculated by software such as *LaserCal*.

For more information about the characterization of frequency standards, please refer to Barnes, J. A. et. al., *Characterization of Frequency Standards.* IEEE Trans. on Instr. and Meas., Vol IM-20, 105-120, (May 1971).